

Accidental Inflation in the Landscape

Jose J. Blanco-Pillado^a, Marta Gomez-Reino^b and Konstantinos Metallinos^a

^a *Institute of Cosmology, Department of Physics and Astronomy*

Tufts University, Medford, MA, US and

^b *Department of Physics, University of Oviedo, Oviedo, Spain.*

Abstract

We study some aspects of fine tuning in inflationary scenarios within string theory flux compactifications and, in particular, in models of accidental inflation. We investigate the possibility that the apparent fine-tuning of the low energy parameters of the theory needed to have inflation can be generically obtained by scanning the values of the fluxes over the landscape. Furthermore, we find that the existence of a landscape of eternal inflation in this model provides us with a natural theory of initial conditions for the inflationary period in our vacuum. We demonstrate how these two effects work in a small corner of the landscape associated with the complex structure of the Calabi-Yau manifold $P^4_{[1,1,1,6,9]}$ by numerically investigating the flux vacua of a reduced moduli space. This allows us to obtain the distribution of observable parameters for inflation in this mini-landscape directly from the fluxes.

I. INTRODUCTION

The idea that our universe has undergone a period of inflation seems to fit the current observations of the cosmic microwave background [1]. However, these observations have not taught us, so far, much about its fundamental origin. It is therefore interesting to analyze the different scenarios that can give rise to inflation within a fundamental theory with the hope that one could identify a distinctive signature that allow us to understand the relevant dynamics that were in play in these early stages of cosmic history.

These ideas have led many people to investigate the cosmological consequences of different high energy models and, in particular, string theory (See, for example, some of the reviews on the subject in [2]). One problem that has plagued this field from the beginning was the necessity of a model of stable compactification that reduces the original $10d$ theory to its $4d$ low energy form. Recently the old Kaluza-Klein idea of flux compactifications [3] has been revived within the string theory set up [4]. These models indicate the existence of perturbatively stable vacua of the theory where we could find ourselves today [5, 6]. Nevertheless it is pretty clear by now that the mechanism of compactification is very far from unique, and it seems very likely that this method of compactifying would lead to a immense landscape of distinct vacua [7, 8]. Some of these vacua would be similar to our four dimensional universe, but others would have very different properties, for example the value of the cosmological constant [7], the low energy physics [9] or even the number of large dimensions [10]. Transitions between these vacua are allowed by a tunneling event where a bubble of the new vacuum is produced in the background of the parent vacuum. This process can continue due to the presence of metastable vacua with positive cosmological constant leading to the picture of an eternally inflating spacetime where all vacua are explored in what has been collectively called the multiverse.

These ideas in models of flux compactification inspired a flurry of papers on new scenarios of inflation within string theory. Most of these studies concentrate on identifying a particular sector of the effective potential for the $4d$ fields that allows for inflation. We can classify these models depending whether the inflaton field is related to the position of a D-brane along the extra dimensions (D-brane inflation [11]) or whether it parametrizes the shape and size of the internal manifold (Modular Inflation [12]).

Modular inflation is a natural idea in any higher dimensional extension of the standard

model since the potential is already present in the construction to fix the moduli fields. On the other hand, it is clear that this could be a very complicated function with many possible forms. In models with fluxes, this effective potential encodes the information about the quantized fluxes that thread the cycles in the internal space. Turning on these fluxes generates a change in energy as a function of the size of the cycles, which in turn is interpreted as potential for these fields. Studying the implications of a particular set of fluxes implies that one is focusing on a single realization of the corresponding potential in the landscape. This has been the approach used in most of the concrete models of inflation in string theory so far. This is a reasonable thing to do since cosmological observations would only allow us to see the last 60 e-folds of inflation¹ which, presumably, would happen within the effective potential with the same set of fluxes.

Furthermore, it is likely that there are many different regions of the landscape that allow for inflation (even within the same model) and if this is the case, we will have to face the question of what is the most likely inflationary scenario on the string theory landscape. The answer to this question will require us to adopt some measure on how to give probabilities to all these inflationary trajectories. This is a hard problem that has been extensively investigated in the last few years and although some progress seems to have emerged from these studies there is not a clear consensus in the literature on how to assign these probabilities (See [16] for some recent discussion on the subject). We will not have anything new to say here about the measure problem and concentrate on another issues where the existence of a landscape can play a significant role, namely, in the fine tuning of the potential as well as the initial conditions required for a successful inflation to occur. This is a first necessary step towards extracting observational predictions in the landscape which would require information about the underlying theory as well as the measure problem [17].

Most models of inflation so far studied in string theory suffer from some kind of fine tuning that, from a purely effective field theory perspective, could be considered quite severe. However, changing the fluxes along the internal manifold will in practice allow the parameters in the effective potential to scan over different values so there could be many different sets

¹ This is not entirely true since it is possible that a tunnelling transition between different vacua could leave its imprint in some of the cosmological observables that we can detect, provided that the total amount of inflation within our bubble is not too large (See [13] or more recently [14, 15]). We will discuss this issue in more detail later on in the paper.

of fluxes that would lead to a particular value for the coefficient in the potential compatible with observations. The most important example of this phenomenon is, of course, the idea that the cosmological constant problem could be solved by the vast numbers of possible vacua that we have in the landscape, so some of them could land on the very narrow observationally allowed region [7]. In this paper we will show that this same idea could help with the fine tuning required to flatten the potential during inflation.

There are several interesting papers in the literature that try to model the complexity of the $4d$ effective potential in the landscape by statistical arguments, see for example [18–21]. In the following, we will compare our methods to some of these other approaches when appropriate.

The outline of the paper is as follows. We first provide, in Section II, an overview of the methods that lead to the effective potential in models of Type IIB flux compactifications. In Section III we show how a simple example of fine-tuned accidental inflation [22] could arise in a toy model for the landscape. In Section IV we explore a small corner of the landscape and find how the relevant parameters of the model are scanned in this mini-landscape and we describe how a natural choice of initial conditions can arise in this models. In Section V we give the distribution of the different observational parameters obtained in our example of the landscape. We conclude in Section VI with some comments and a general outlook.

II. REVIEW OF FLUX COMPACTIFICATIONS

Most models of inflation based on the evolution of moduli fields use, as a starting point, the low energy description of the supersymmetric string theories, namely a supergravity theory. In our case we will focus on Type IIB compactifications to four dimensions on a Calabi-Yau (CY) orientifold, which reduces the scalar field theory to an $\mathcal{N} = 1$ supergravity action for the moduli fields. In the following we will briefly describe the ingredients necessary to specify the $\mathcal{N} = 1$ theory for our moduli fields.

A. $\mathcal{N} = 1$ SUGRA actions

The effective action of $\mathcal{N} = 1$ supergravity with n chiral fields, Φ_i , will be fixed, for the purposes of this paper, once one specifies two different functions: a holomorphic superpo-

tential $W(\Phi_i)$, and a real Kahler potential $K(\Phi_i, \Phi_i^\dagger)$. In terms of these functions the F-term scalar potential is given by²:

$$V_F = e^K \left(\sum_i K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right) \quad (1)$$

where

$$D_i W = \partial_i W + \partial_i K W \quad ; \quad D_{\bar{j}} \bar{W} = \partial_{\bar{j}} \bar{W} + \partial_{\bar{j}} K \bar{W} . \quad (2)$$

and the kinetic terms of the fields are computed from the Kahler potential using,

$$\mathcal{L}_{kin} = K_{I\bar{J}} \partial_\mu \Phi^I \partial^\mu \bar{\Phi}^{\bar{J}} . \quad (3)$$

There may be, in general, other terms in the effective potential for the fields that could be included in a supergravity action, but this simple description would be sufficient for the type of models that we will describe below.

1. Kinetic terms for the moduli

One can obtain the information for the kinetic terms for the moduli fields by performing a dimensional reduction of the 10d theory where the internal manifold is a Calabi-Yau threefold [23]. Following this procedure one identifies three different type of fields, the complex structure moduli, the Kahler moduli and the axion-dilaton field.

In the rest of this section we will briefly describe each of these sectors of the scalar field space and their kinetic terms.

- Complex Structure

The Kahler potential for the complex structure moduli can be calculated to be [23],

$$K_{cs} = -\log \left(i \int_M \Omega \wedge \bar{\Omega} \right) \quad (4)$$

² We fix $M_P = 1$ and use the standard notation $F_i = \frac{\partial}{\partial \Phi_i}$, $F_{\bar{j}} = \frac{\partial}{\partial \Phi_{\bar{j}}^\dagger}$, \dots , with F being any function of the fields. Also note that indices are lowered and raised with the Kahler metric $K_{i\bar{j}}$ and its inverse $K^{i\bar{j}}$.

where Ω denotes the holomorphic three form and the integral is performed over the Calabi-Yau threefold M . We can also rewrite the expression above as,

$$K_{cs} = -\log (-i\Pi^\dagger \cdot \Sigma \cdot \Pi) \quad (5)$$

where we have introduced the symplectic matrix, Σ , given by,

$$\Sigma = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix} \quad (6)$$

as well as the period vector,

$$\Pi(w) = (w^a, F_a) \quad (7)$$

whose components are defined in terms of integrals of the holomorphic three-form Ω over the tree-cycles, A^a, B_a , namely,

$$w^a = \int_{A^a} \Omega \quad F_a = \int_{B_a} \Omega . \quad (8)$$

On the other hand, F_a can be computed in terms of the so-called prepotential F by the relation, $F_a = \partial_a F(w)$. The complex structure moduli fields z_i are then obtained via the normalization condition, $z_i = \frac{w^i}{w^0}$ so, having the expression for the prepotential in terms of the w^i coordinates one can write the Kahler potential as a function of the complex structure fields, $K_{cs}(z_i)$.

We will later describe in detail the complex structure moduli space of a particular CY where all these definitions will become more clear.

- Kahler moduli

The Kahler potential for the Kahler moduli is given by,

$$K_K = -2\log(\mathcal{V}) , \quad (9)$$

where \mathcal{V} denotes the volume of the Calabi-Yau in string units and can be written as,

$$\mathcal{V} = \int_M J^3 = \frac{1}{6} \kappa_{jk} t^i t^j t^k \quad (10)$$

where J is the Kahler form of the CY and the t^i fields denote the sizes of its 2-cycles. We can also, for later convenience rewrite everything in terms of the volumes of the four-cycles, τ_i , by the following expression,

$$\tau_i = \partial_{t^i} \mathcal{V} = \frac{1}{2} \kappa_{ijk} t^j t^k. \quad (11)$$

So far we have only discussed four dimensional massless fields whose origin was a deformation of the internal geometry. But it is clear that there will be more massless scalar fields coming from the compactification of different p-forms present in the ten dimensional theory. In fact, one can show that there are the same number of axionic fields coming from the compactification of the Ramond-Ramond 4-form that pair up with the Kahler moduli fields to create the complexified Kahler moduli,

$$T_i = \tau_i + i\theta_i, \quad (12)$$

which are the fields that we will include in the $\mathcal{N} = 1$ supergravity description of the four dimensional theory. In this paper we will be mainly interested in a simple toy model of a single Kahler moduli with Kahler function,

$$K_K = -3 \log (T + \bar{T}) . \quad (13)$$

- Dilaton

Finally, there is another important component of the four dimensional theory which is given by a complex field composed of the zero mode of the dilaton (ϕ) and the zero mode of the axion field (the Ramond-Ramond zero form, C_0) already present in ten dimensions, namely,

$$\tau = C_0 + i e^{-\phi}, \quad (14)$$

such that in the low energy four dimensional theory we have a kinetic term for this field coming from the Kahler potential of the form,

$$K_d = -\log(-i(\tau - \bar{\tau})) . \quad (15)$$

It is important to keep in mind that the dilaton controls the string coupling constant, so our final minima should be stabilized at small values of $g_s = e^\phi$ which in turn means that $Im(\tau) > 1$.

One can then obtain the final expression for the Kahler function for all the fields present in the low energy theory as the sum of all three types of fields, namely,

$$K = K_{cs} + K_K + K_d . \quad (16)$$

2. *Superpotential induced by fluxes.*

So far, we have only discussed the structure of the field space of the four dimensional moduli, so at this level the fields remain massless. The new ingredient recently added to the string theory picture is the introduction of fluxes threading the internal cycles of the CY. These fluxes will give rise to a four dimensional energy density that depends on the size of the internal cycles, in other words, they will induce some potential to the complex structure moduli [5].

The expression for this potential is obtained in the $4d$ supersymmetric theory once we know the superpotential. This superpotential in models of flux compactification can be shown to be of the form [24]

$$W_{GVW} = \int_M G_3 \wedge \Omega = \int_M (F_3 - \tau H_3) \wedge \Omega , \quad (17)$$

where F_3 and H_3 denote the three form field strengths present in the theory and Ω is the holomorphic three form of the CY. It is clear from our previous definitions that this superpotential will depend on the dilaton as well as the complex structure moduli therefore generating a potential for those fields.

On the other hand, the potential found this way does not depend on the Kahler moduli so it does not help to stabilize this sector of the theory. In order to do this one would need to include a new piece in the superpotential that depends on T_i . A model of this type was first discussed in [6].

B. **The KKLT construction**

In [6] it was argued that non-perturbative terms would be introduced in the superpotential if there is a gaugino condensation on a sector of the theory. The interesting point about this new term in the superpotential is that it would depend on the Kahler moduli

therefore allowing for the possibility of stabilizing these fields as well. A typical term in the superpotential would have the form,

$$W_{NP} = \sum_i A_i e^{-a_i T_i} . \quad (18)$$

The KKLT construction is based on the idea of a 2 step process. In the first part one imagines the complex structure to be fixed by the fluxes in a way that we described above such that their moduli fields are stuck to their minima. The value of the superpotential at the minima of the complex structure fields is denoted by W_0 and it is added to the non-perturbative term coming from the gaugino condensation. Assuming a superpotential contribution from the gaugino condensation of the racetrack type and the presence of a single Kahler moduli for simplicity, the authors of [6] arrived at a total superpotential of the form

$$W = W_0 + A e^{-aT} + B e^{-bT} . \quad (19)$$

The potential generated in this way can have several minima and therefore fix the Kahler moduli. On the other hand, most of the minima that one would find using this potential would have a negative value of the cosmological constant. In particular, it is easy to see that in supersymmetry preserving minima they would be either Minkowski [25] or anti-de Sitter [6]. We must therefore *uplift* these minima to be compatible with current cosmological observations. These considerations lead them to introduce a term of the form

$$V_U = \frac{D}{\mathcal{V}^2} , \quad (20)$$

where \mathcal{V} denotes as before the volume of the CY, and can be written in terms of the Kahler moduli T_i .

Putting all these various ingredients together, one can write the effective potential for a single Kahler moduli with Kahler function given by $K = -3 \log (T + \bar{T})$ in the form,

$$\begin{aligned} V(X, Y) = & \frac{1}{6X^2} [e^{-2(a+b)X} (aA^2(aX+3)e^{2bX} + bB^2(bX+3)e^{2aX}) \\ & + e^{-(a+b)X} (AB(2abX+3(a+b)) \cos(Y(a-b))) \\ & + 3aAe^{bX} (W_{0X} \cos(aY) - W_{0Y} \sin(aY)) \\ & + 3bBe^{aX} (W_{0X} \cos(bY) - W_{0Y} \sin(bY))] \\ & + \frac{D}{X^2} \end{aligned} \quad (21)$$

where we have separated the real ($\text{Re}T = X$) and imaginary parts ($\text{Im}T = Y$) of the field as well as the superpotential ($\text{Re}W_0 = W_{0X}$) ($\text{Im}W_0 = W_{0Y}$). Using this potential one can stabilize the Kahler moduli as well in a non-supersymmetric de Sitter vacua compatible with our current observations [6].

This is an interesting construction and there are, by now, many different variations on how to obtain a phenomenologically viable vacua in this way. Furthermore, as we discussed in the introduction finding a stable compactification allows people to think of new ways to describe the evolution of the Early Universe in these models. In particular, one may wonder if the same potential that induces the compactification could be responsible for the energy density of inflation. Having a computable potential away from its minimum permits us to calculate not only the dynamics of the scalar fields in this potential but observational parameters such as the scale of inflation, the amplitude of perturbations and its spectral index [12]. There are many realizations of this idea in the literature, but in the following we would like to concentrate on a simple model of inflation, accidental inflation [22].

III. ACCIDENTAL INFLATION

This is an inflationary scenario that is easily embedded in a KKLT model of flux compactification. The idea is to look for an approximate inflection point in the Kahler field sector of the potential that allows for sufficient inflation. In [22] the authors found that a region of this type could be generated along the real part of the complex Kahler field in the simplest models with a racetrack type potential.

However, one can argue that the model is quite constrained by what is perhaps more than one type of fine tuning. Firstly, in order for the inflection point to lead to a sufficient number of e-folds the potential has to be finely tuned so that it becomes flat enough around the inflection point. The hope is then that this coincidence can happen at some point of the large parameter space available to string theory, hence the name *accidental inflation*.

On the other hand, these type of models also suffer from an overshoot problem [26]. This is a somewhat generic problem in models where the inflationary region is small because one has to make sure that the field arrives to this point in space with sufficiently low velocity so it can stay in the slow roll inflation region for sufficient amount of time. This problem is related to the question of what are the natural initial conditions for the fields before inflation.

This is of course an important question for most of models of inflation but in these kind of inflection point scenarios this is an especially relevant issue.

A particular example of accidental inflation can be obtained by choosing the parameters

$$A = \frac{1}{145}; \quad B = -\frac{1}{145}; \quad a = \frac{2\pi}{580}; \quad b = \frac{2\pi}{600}; \quad (22)$$

$$W_0 = 1.01796 \times 10^{-4} + 3.1034287 \times 10^{-5}i; \quad D = 6.0614989 \times 10^{-12}.$$

The idea behind this choice of values for A, B, a and b is that our set of parameters leads to an inflection point whose 3rd derivative is relatively small so one can have a region (not just a point) in field space that satisfies the slow roll condition. This is not an important fine tuning since it can be achieved in a large region of the parameter space but as we will see this becomes quite relevant for our conclusions. We use a W_0 superpotential with a real and imaginary part in order to avoid overshooting the overall minima of the potential, in other words to avoid decompactification. The effect of this complex superpotential is to displace the value of the Y component of the field at the inflection point relative to the overall minimum. One then has a curved trajectory in field space that allows one to reduce the kinetic energy in the X direction and avoid decompactification. We show in Fig. (1) the field trajectory around the inflection point and the subsequent evolution in a flat Friedmann-Robertson-Walker (FRW) universe where we have chosen as our initial conditions a point in field space at the beginning of the slow roll region. We note that even though the full trajectory is curved, the relevant part for inflation happens near the inflection point so the predictions of this model are closely related to single field inflection point models [27–29]. The total number of e-folds for this case is 165 and the amplitude of perturbations as well as the spectral index are compatible with observations.

One can easily see in this example the problems associated with the fine tuning that we were discussing earlier. Changing the parameters in the potential by a small amount destroys the nice properties of the inflection point and the potential would not give rise to any number of e-folds. On the other hand, choosing the initial conditions far away from the inflection point also has an important effect since the fields pick up too much kinetic energy by the time they arrive at the inflection point to have enough inflation, or even worse, they do not approach that point at all and run directly to the global minimum.

It is clear then that one would like to investigate the possibility of ameliorating some of

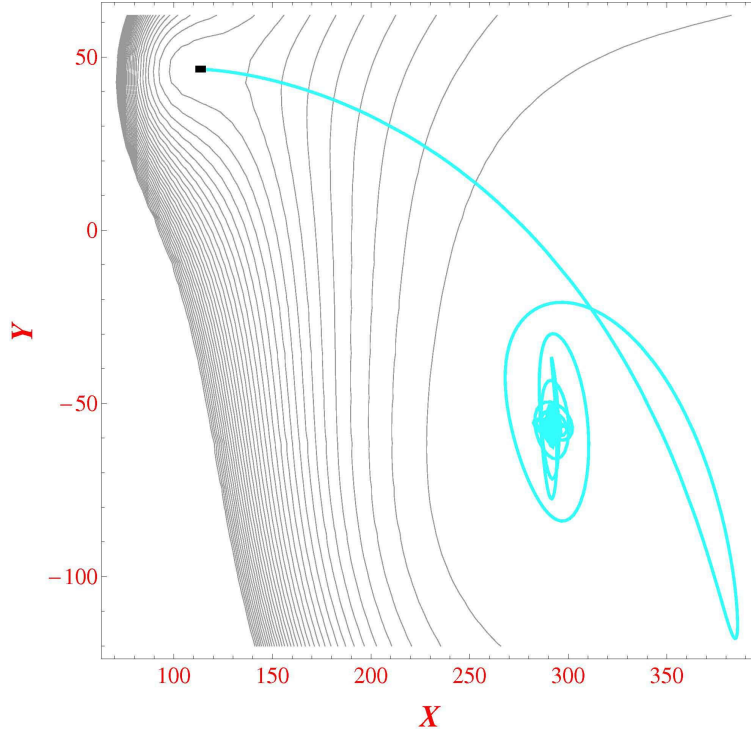


FIG. 1: Example of Accidental Inflation. We show the inflationary trajectory superimposed over the contour plot of the effective potential in the $X - Y$ plane. We mark in black the small region of the trajectory where the slow roll conditions are satisfied.

these fine tunings. In the following we will explore some ideas in the context of this simple model that show how the existence of a landscape could help with both these issues³.

IV. ACCIDENTAL INFLATION IN A CORNER OF THE LANDSCAPE

As we described in the introduction, string theory provides us with a way to scan different values of the parameters of the low energy effective action. This has important consequences for our understanding of what can be considered fine tuning of the moduli potential as well as the predictions for the observable parameters for inflation.

One could try to investigate this effect by assuming that the parameters of the low energy

³ Note that one can also reduce the severity of this fine tuning problem assuming a particular family of measures [22] that would give a overwhelming weight to any trajectory that inflates compared with the other ones. As described in the introduction, we will not consider these types of measure problems in this paper.

theory would vary over some range of values in the landscape. In our case this would mean, in practice, to allow the first and third derivatives of the potential around the inflection point to vary with some prescribed distributions, similarly to what was done for D-brane inflation in [19, 20]. On the other hand it is clear that these parameters in the moduli potential are not fundamental themselves but are computed in terms of the fundamental ingredients that vary in a quantized manner over the landscape. In models of flux compactifications based on Type IIB string theory, one specifies the 4d moduli potential once the fluxes along the three cycles of the internal manifold are fixed. This suggests that we should study the mapping between the various sets of fluxes and the values of the parameters in the low energy theory to obtain a more accurate description of the distribution of the parameters of the potential in a real landscape.

Given a set of the fluxes one can obtain, following the prescription given above, the value of the complex structure moduli which, in turn, fixes the value of the superpotential, W_0 . This means that one can think of this parameter in the potential as being scanned over the landscape. The key point is then to realize that one can control the slope of the potential around the inflection point by choosing W_0 appropriately, leaving all the other parameters fixed. This is important for our inflationary models since by decreasing the slope of the potential at the inflection point one increases the number of e-folds in that region. In the following we will use a particular CY to study in detail the distribution of values of W_0 in a mini-landscape and to see how this affects the probability distribution of the number of e-folds.

There are, of course, other important fine tunings of this modular potential that one has to address in order to obtain a successful model of inflation in string theory. In particular one should fix the global minimum of the potential at a vanishing value of the cosmological constant which in turn requires us to tune the *uplifting* parameter D , so we should also consider this parameter to be scanned over the landscape. This is a much more serious fine tuning than the one required for inflation to happen. We will not try to address these two issues at the same time and in the following we will assume that some other sector of the landscape is responsible for the extreme fine tuning of the parameter D so we can basically consider it a continuous parameter that can be fixed to have an appropriate value of the potential at its global minimum.

A. The $P^4_{[1,1,1,6,9]}$ Calabi-Yau

In order to investigate in detail the implications of the existence of a landscape we will look at a particularly simple model for the complex structure fields on one of the best known Calabi-Yaus, the orientifold of $P^4_{[1,1,1,6,9]}$. This CY threefold has 2 Kahler moduli and 272 complex structure moduli. However we will restrict ourselves to a 2 dimensional slice of the complex structure moduli that can be obtained by imposing a particular symmetry on the manifold. (See [30, 31] for more details on this manifold). This is of course a very small number of complex structure fields and it is by no means representative of a typical CY. However we choose to work with this relatively small number of moduli so we can explicitly perform the numerical calculations in a reasonable amount of time. For other numerical investigations of the complex structure sector see [33].

1. Complex Structure

Following the KKLT procedure described earlier, we will first find the solutions of the supersymmetric equations for the complex structure moduli, and the dilaton, namely:

$$D_I W = 0 \quad ; \quad D_\tau W = 0 \quad (23)$$

where $I = 1 \dots h^{2,1}$. To solve these equations we need the information of the internal geometry, in other words, we need to find the Kahler function for the complex structure moduli. As described in the previous sections the first step in this procedure is to obtain the prepotential which in our case was computed in [30–32],

$$F = (w^0)^2 \mathcal{F} = (w^0)^2 \left(\frac{1}{6}(9z_1^3 + 9z_1^2 z_2 + 3z_1 z_2^2) - \frac{9}{4}z_1^2 - \frac{3}{2}z_1 z_2 - \frac{17}{4}z_1 - \frac{3}{2}z_2 + \xi \right) \quad (24)$$

where we use the following normalization for the complex structure fields, $z_i = \frac{w^i}{w^0}$, and we have also defined

$$\mathcal{F} = \frac{1}{6}(9z_1^3 + 9z_1^2 z_2 + 3z_1 z_2^2) - \frac{9}{4}z_1^2 - \frac{3}{2}z_1 z_2 - \frac{17}{4}z_1 - \frac{3}{2}z_2 + \xi. \quad (25)$$

With this prepotential and using the normalization of $w^0 = 1$, we can now compute the vector periods,

$$\Pi = (1, z_1, z_2, 2\mathcal{F} - z_1\mathcal{F}_1 - z_2\mathcal{F}_2, \mathcal{F}_1, \mathcal{F}_2). \quad (26)$$

Using Eqs. (5) and (6), the Kahler function for the complex structure moduli in terms of z_1, z_2 is given by,

$$K_{cs}(z_1, z_2) = -\log [i ((z_1 - \bar{z}_1)(\mathcal{F}_1 + \bar{\mathcal{F}}_1) + (z_2 - \bar{z}_2)(\mathcal{F}_2 + \bar{\mathcal{F}}_2) - 2(\mathcal{F} - \bar{\mathcal{F}}))] \quad (27)$$

which in our case becomes, using $z_i = X_i + iY_i$,

$$K_{cs}(z_1, z_2) = -\log [4Y_1(3Y_1^2 + 3Y_1Y_2 + Y_2^2) - 4i\xi] \quad (28)$$

following [31] we use $\xi = -1.3i$.

The superpotential generated by the fluxes can be computed using the expression

$$W_F = \frac{1}{(2\pi)^2\alpha'} \int_M (F_3 - \tau H_3) \wedge \Omega \quad (29)$$

which can also be written in terms of the dilaton and complex structure as well as the integer fluxes through the A and B 3-cycles as,

$$W_F = \sum_{i=0}^2 [(f_A^i - \tau h_A^i)F_i - (f_B^i - \tau h_B^i)z_i] \quad (30)$$

where we have defined

$$f_{A,B}^i = \frac{1}{(2\pi)^2\alpha'} \int_{A^i,B^i} F_3 \in Z, \quad (31)$$

$$h_{A,B}^i = \frac{1}{(2\pi)^2\alpha'} \int_{A^i,B^i} H_3 \in Z. \quad (32)$$

Fixing the flux integers and using the definitions of the Kahler function, Eq. (27), and the superpotential, Eq. (30) we can now obtain the values of the complex structure fields and the dilaton at a supersymmetric minima by solving Eqs. (23). Plugging the solutions back into the expression for the superpotential we can easily compute W_0 at that point.

We have followed this procedure for a large number ($\approx 10^9$) of combinations of the fluxes and observe that the values of W_0 seem to be uniformly distributed over a large area (of the order of 10^4) of the complex plane. We show in Fig. (2) a small sample of 10^4 randomly selected values over a small region that clearly demonstrates this point. This is a similar result to the one obtained in [34] and it will have important consequences for us later. Note that not all these points will end up being minima of the full potential, since some of them may be stable supersymmetric AdS critical points which will be transformed into saddle points by the uplifting procedure. One can, of course, resolve this issue by identifying the true minima of the full potential, but for the moment we will be content with this KKLT procedure. Our results should not change even if the fraction of these points that are true minima is small.

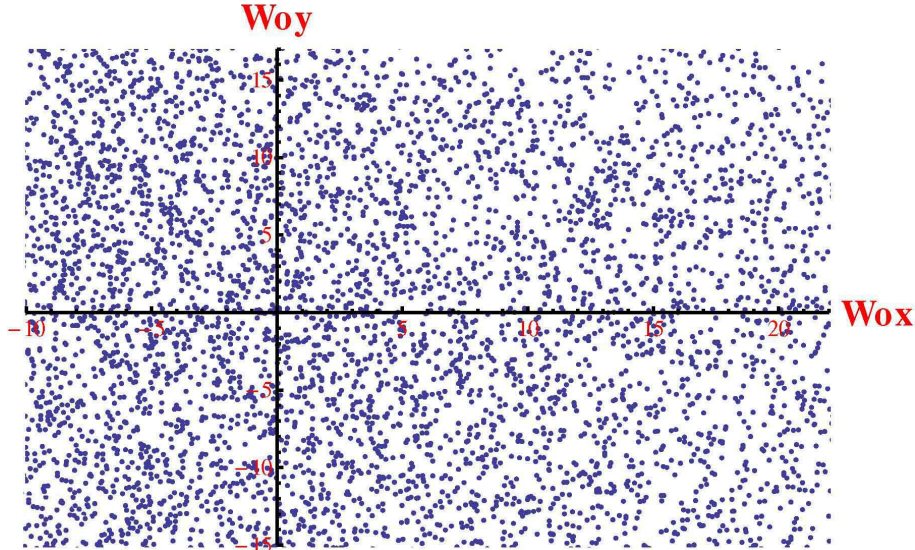


FIG. 2: Values of W_0 for a small sample of supersymmetric solutions in the landscape generated by turning on six fluxes over the $P^4_{[1,1,1,6,9]}$ manifold.

2. The Kahler moduli

We are mainly interested in the effect of the complex structure moduli in our inflationary model so in order to simplify our analysis we will focus on a very simple toy model for the Kahler fields whose Kahler function is given by Eq. (13). We will also assume the presence of a non-perturbative potential as well as an uplifting term as described in the KKLT constructions above.⁴

Finally we will also consider the case where the parameters of the non-perturbative superpotential A, B, a, b do not have a strong dependence on the complex structure moduli and take them to be constant for all the values of the fluxes. This is likely to be true for many of our vacua since we have not seen large changes of the values of the complex structure moduli over the scanned vacua. Even if this assumption is violated in some cases, it will be true for many of the vacua and our conclusions are likely to hold.

⁴ Note that one could take the Kahler moduli specific for our CY manifold, but this would make the inflationary model considerably more complicated. We will come back to this issue as part of our future work.

B. Exploring a mini-Landscape

In the previous section we have shown an example of a successful inflationary model where all the observational constraints of the model were satisfied. In particular one could find a region of the potential that allowed for a large number of e-folds. This was achieved by a quite severe fine tuning of the superpotential around $W_0 = 1.01796 \times 10^{-4} + 3.1034287 \times 10^{-5}i$. This implies that the possible values of W_0 to make this model work would be confined to a tiny area in the W_0 complex plane of the order of 10^{-17} . Taking into account that the distribution of vacua on the W_0 plane seems to be uniform, we can estimate the fraction of vacua that would be found in this preferred region if one was to generate a large number of flux combinations. Following this calculation we can easily see that we will not be able to explore the landscape finely enough with our mini-landscape in a reasonable amount of time.

On the other hand, the main reason to go to these small values of W_0 was to satisfy all the observational constraints, in particular the idea that the scale of inflation should be small to accommodate the amplitude of perturbations. In the following we will sacrifice this requirement in order to demonstrate in a concrete example some of the ideas presented earlier about the fine tuning necessary to achieve large number of e-folds. We therefore explore another region of the parameters where one still needs a considerable degree of fine tuning but can be accessed with our mini-landscape with a sample of generated vacua of a much more manageable size.

1. Specific Example

We start by picking a generic set of values for A, B, a and b in a region of parameter space that leads to a large value of W_0 . We imagine that these parameters are fixed by the effective theory of a hidden sector. We take the values,

$$A = 5; \quad B = -10; \quad a = \frac{2\pi}{100}; \quad b = \frac{2\pi}{290} . \quad (33)$$

One can now obtain a range of values for W_0 and D such that the potential would have a near inflection point at some positive value of the potential, as well as a global minimum with vanishing cosmological constant. This requires W_0 to be within a small area in the complex plane of the order of $\approx 10^{-4}$. This is of course a fine tuned value from the low

energy perspective, but the question we would like to address is if one should expect to find several vacua within this narrow strip of values or not, taking into account the existence of the landscape.

In order to address this question we find the values for the superpotential at the supersymmetric minima using the machinery described above. We do this by first generating a large number (of the order of 10^9) of combinations of fluxes and solving the supersymmetric conditions Eqs. (23). Armed with the values of the complex structure and dilaton at their minima, we calculate the superpotential at those values and identify the ones that land within our region of interest. Following this procedure, we were able to find ~ 50 combinations of fluxes with the correct values of the superpotential. We show in Fig. (3) the region of W_0 required to obtain inflation in this model as well as the location of the particular values of the flux vacua that we found following the steps described above.

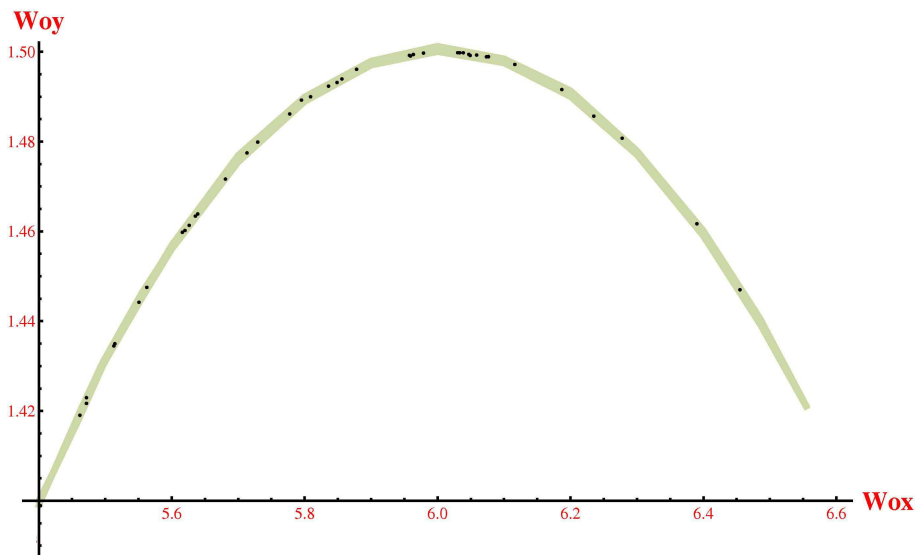


FIG. 3: We show in this figure the small shaded area in the W_0 plane consistent with more than 60 e-folds of inflation for the parameters given in Eq. (33). Each of the points in this region represent a particular combination of fluxes that leads to this value of W_0 .

We note that the number of vacua we found in this region is actually in good agreement with the assumption that the superpotential scans uniformly the values of the complex plane within the area between roughly $W_X = (-100, 100)$ and $W_Y = (-100, 100)$. This suggests that one can follow this calculation for other cases that can not be explicitly done by numerical calculation and therefore one can assume that it is likely that the small fine-

tuned region of the W_0 is actually achieved by quite a large set of flux vacua.

The values of the fluxes are not totally arbitrary, they are constrained by several technical requirements. The first one is due to the tadpole condition, which basically imposes that the total $D3$ brane charge be zero. In practice, this creates an upper limit on a combination of the fluxes over the internal manifold. Here we will follow Douglas et al. [31] and will only consider the sets of fluxes restricted to the condition,

$$N_{flux} = \frac{1}{(2\pi)^4 \alpha'^2} \int_M F_3 \wedge H_3 < 350 . \quad (34)$$

Also, we will only keep fluxes whose vacua are stabilized at a large value $\text{Im}(\tau)$. This will make sure that we will find these vacua in the weak coupling regime. Similarly, in order to disregard instanton corrections in the prepotential in Eq. (24) we will only use fluxes that lead to $\text{Im}(z_i) \geq 1$ for $i = 1, 2$. Finally we are only considering fluxes that are inequivalent, meaning our set of the fluxes are not related by a $SL(2, Z)$ symmetry transformation. This prevents us from overcounting the number of fluxes in the region of interest for us.⁵

2. Initial Conditions

In order to make predictions for the observable parameters of this landscape one has to consider some initial conditions. As we argued above choosing these type of values of the parameters relaxes the extreme fine tuning of the initial conditions in this model, since we now have a flat section of the potential where the slow roll conditions can be satisfied, but this does not explain why should the universe start at all close to the inflationary plateau in the vast region of field space.

Here we point out that the idea of the landscape also helps us understanding why this happens. Let us think for a moment on the other flux vacua in the theory, the ones that could be the parent vacua for the one that we find ourselves today. It is clear that there will be many other combinations of fluxes that give a nearby value of W_0 . The important point is to realize that many of those other values of the superpotential will turn the inflection point

⁵ These requirements reduce the number of valid minima from the naive calculation based on the uniform distribution of vacua on the W_0 plane since, sometimes, many of these vacua violate some of these restrictions. We have not seen however that this would produce any voids in the W_0 distribution, so the argument is still basically valid.

in the Kahler moduli potential into a local minima. This makes it possible for the fields to be stuck on a particularly interesting value in the parent vacua, somewhat near the inflationary plateau of the daughter vacuum ⁶. One can imagine that the flux changing transition would mainly affect the complex structure fields and would not have a great impact on the values of the Kahler moduli. This is a reasonable assumption since, after all, this transition could happen in a local part of the internal geometry and presumably would not change the overall volume of the internal manifold (parametrized by the field X) by a whole lot.

Having this process in mind, one should consider that each of these satisfactory *daughter* vacua can have in principle many predecessors that can give rise to it, (*her parent vacua*). The idea is then that the initial conditions for the field evolution in the daughter vacuum should be set by the conditions in the predecessor. This suggests that we should look for the form of the effective potential outside of the region of the W_0 that gives an inflection point inflation and identify the minima of that other vacua. In order to do this in our example, we choose one particular daughter vacuum and investigate it in more detail, assuming that we only change the value of W_0 , in other words we will leave the parameter D constant.

For example, let us consider the following set of flux integers,

$$\begin{aligned} f_A^i &= (17, -2, 0) \quad ; \quad f_B^i = (5, -47, -12) \quad ; \\ h_A^i &= (-2, -4, 4) \quad ; \quad h_B^i = (44, 22, 3) . \end{aligned} \tag{35}$$

With these fluxes one can show that the solution of the supersymmetric Eqs. (23) for the complex structure moduli and the dilaton take the values,

$$z_1 = -0.749 + 0.991i , \tag{36}$$

$$z_2 = 2.043 + 0.977i , \tag{37}$$

$$\tau = -1.28 + 2.87i . \tag{38}$$

Using these results we obtain the superpotential at this point,

$$W_0 = 5.87805764 + 1.49611588i , \tag{39}$$

while the uplifting parameter in this case should be,

$$D = 0.0642811355 . \tag{40}$$

⁶ Similar ideas were also studied in the context of D-brane inflation in [21].

Taking all these values into account we use the scalar potential in Eq. (21) to find the inflection point in the Kahler moduli fields at,

$$T = 7.8623431 + 14.2923151i , \quad (41)$$

as well as the global minimum which in this case is situated at

$$T = 66.785459 - 15.343517i . \quad (42)$$

One can show that this potential qualifies as a successful daughter vacuum leading to roughly 115 e-folds of inflation. Changing slightly the value of the superpotential from this one would make the inflection point region steeper or transform it into a local minimum. Thinking in terms of the complex W_0 one can see that there is a relatively large area where one would find a de Sitter local minimum.⁷

On the other hand each set of fluxes gives, at the supersymmetric minimum, a value for the superpotential with a different complex phase and therefore it shifts the inflection point or the minimum in the $X - Y$ field space. We show in Fig. (4) the contour plot of the potential for the set of parameters of our *daughter* vacuum together with the location of a few hundred de Sitter minima that we found by varying the combinations of the fluxes. We consider any of these points a good location for the initial conditions for the interior of the *daughter* bubble that forms as a result of the quantum tunneling event. We note that these are not nearby vacua in the sense of a normal metric on the space of fluxes. In fact, some of these vacua may be away from our daughter vacua by changes in several fluxes. It would be interesting to study the distribution of decay rates for this set of vacua along the lines of ([35–38])⁸. This is important since it enters the final calculation of the probability distribution of any observable in the multiverse [17]. We leave this important issue for future work.

This picture suggests that many of the predecessor vacua for this model would be situated near the inflection point for our daughter vacua. This does not solve all our problems, since even if we start our cosmological evolution from those points, we will still have to face the

⁷ We will not consider the AdS minima as possible parent vacua since they would likely be collapsing before they have time to tunnel to other flux vacua.

⁸ One should also consider other possible ways to induce multiple flux transitions that may be relevant here. See for example [39, 40].

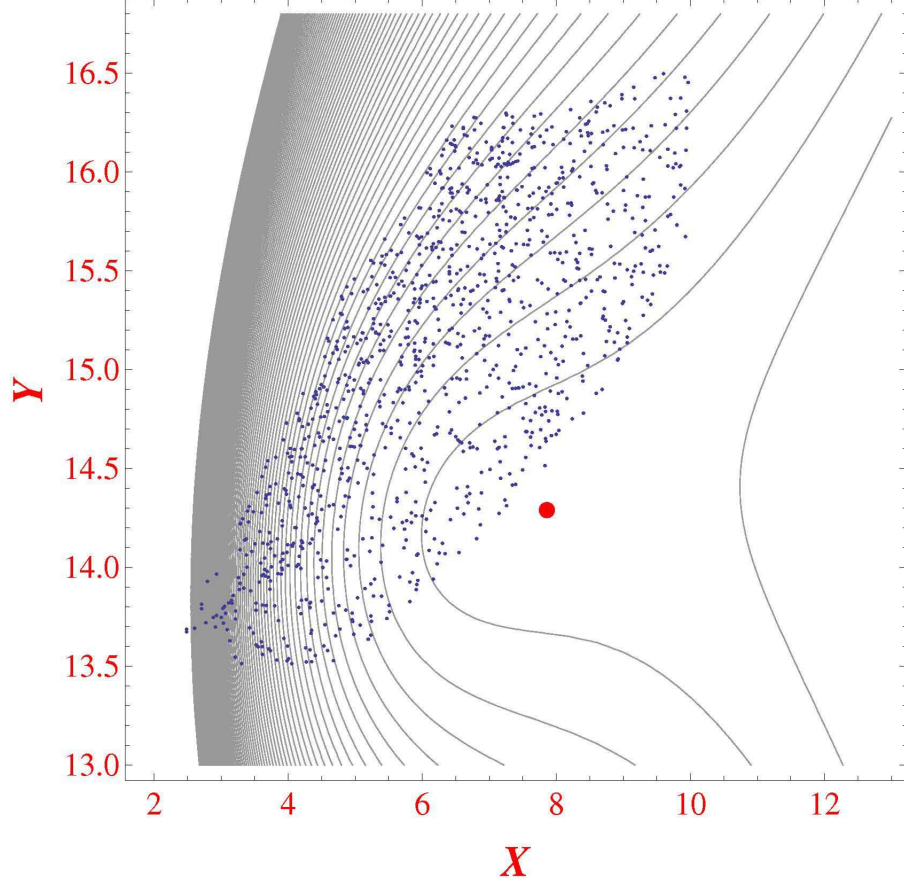


FIG. 4: Plot of the location of the de Sitter parent minima around the inflection point (big red circle) of the daughter vacuum in the $X - Y$ field space. We show in the background the contour plot of the potential for the daughter vacuum case.

overshooting problem. Here we argue that the idea that the inflationary regime in our past was initiated by a flux changing transition also helps with this problem.

It was pointed out in [14, 41, 42] that the presence of a curvature dominated regime in the early stages of the interior of a newly created bubble could help solving the overshooting problem by gently depositing the fields over the inflationary plateau. The situation is more complicated in our case, since we have these parent vacua scattered over the $X - Y$ plane which seems to make the problem of dynamically finding the inflection point a little bit harder.

In order to investigate these ideas we take a large number of de Sitter parent vacua for our case and find, using the equations of motion given in the Appendix, the evolution of the Kahler moduli in the open FRW universe inside of the bubble. We see that even though

the initial point is in some cases far away from the inflection point, the fields roll towards it without overshooting it. This is due to a combination of effects. The first one is the one that we described earlier, the help of friction coming from the fact that the universe is open. The second effect is the evolution of the fields along the perpendicular direction, Y . This evolution allows for some dissipation of the energy stored in the initial conditions and helps the fields to arrive at the slow roll region without so much kinetic energy. The result is an attractor-like behaviour towards the inflection point that is easily seen in Fig (5). This is an important effect since it will increase the range of possible initial conditions that one could take in order to have certain number of e-folds.

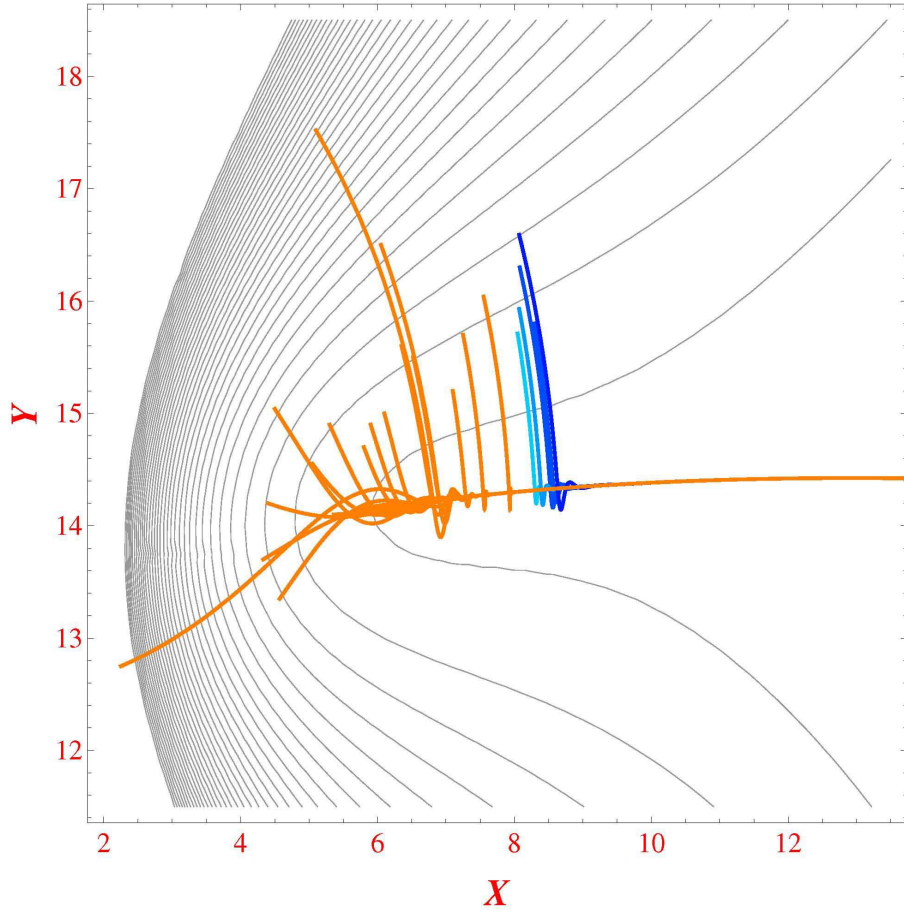


FIG. 5: Plot of a small number of inflationary trajectories for different initial conditions around the inflection point of the daughter vacuum. All the trajectories converge to the same path at the inflection point demonstrating the attractor-like behaviour in our model.

3. Distribution of the number of e-folds

One of the interesting questions we can address in this mini-landscape is what is the distribution of the number of e-folds within the inflationary *daughter* vacua. This was studied for a simple model of the landscape in [14] where it was calculated to be a $1/N^4$ distribution. We would like to understand the similar situation in our case taking into account the inflationary daughter vacua found in Fig. (3). To calculate this distribution, we want to consider the two effects present here, the fact that the effective potential changes with the value of W_0 as well as the possible effect of the distribution of the initial conditions.

We investigate this by looking at the evolution of the fields in each of the realizations of W_0 by using some random initial conditions near the inflection point ⁹ as well as the assumption of an open universe. We show in Fig. (6) the histogram of the number of e-folds for this set of vacua.

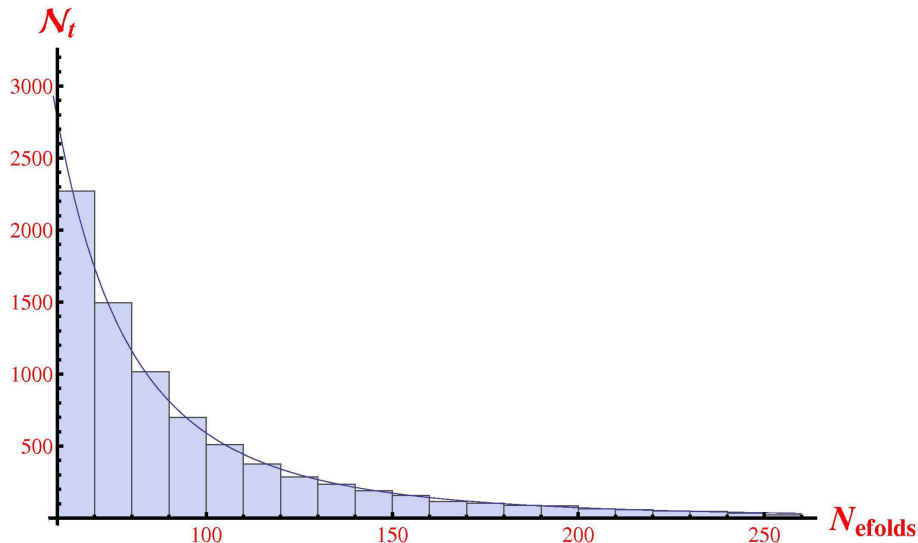


FIG. 6: Distribution of the number of e-folds. We show the histogram of the number of trajectories (\mathcal{N}_t) as a function of the number of e-folds, (N_{efolds}).

The results are well approximated by a $1/N^3$ distribution. One can explain this behavior observing that the distribution of the first derivatives at the inflection point is flat in

⁹ We could, in principle, use the exact location of the de Sitter vacua by calculating the position in each case, but we simplify things a little bit here by taking random initial conditions since the distribution in field space is pretty homogeneous.

this ensemble of vacua and assuming that, due to the attractor like behaviour, the initial conditions do not play a significant role in this distribution. This is a similar result to the one obtained in [19, 20] although in our case we have obtained this distribution directly from the fluxes, and it is not an assumption about the distribution of values of low energy parameters. We give the details of this derivation in a one dimensional toy model in the Appendix.

If this was the only observable prediction of this landscape we would be tempted to argue that it is quite likely to see a small amount of curvature in the universe today, since large numbers of e-folds are hard to achieve inside of our bubble universe. This was first discussed in [14] in a simple toy model for the landscape. (See also the discussion in [43]).

The situation is more complicated in our case if we consider the constraints obtained from the cosmological perturbations associated with the inflaton field. However, we will not discuss the distribution of values of these other observables with the present set of parameters since we are considering a corner of the landscape where the scale of the potential is too high. Remember that this was the prize we had to pay in order to investigate actual vacua of the complex structure minima directly from the fluxes. In the following section, we will return to our original example where we do not have this problem.

V. OTHER OBSERVABLE PARAMETERS IN THE LANDSCAPE

We can now extrapolate the results of the previous section to other regions of the landscape that we can not directly access numerically since the number of required vacua that we would need to explore would be enormous. In particular, we can investigate the dependence of other observational parameters like the amplitude of perturbations as well as the spectral index in the phenomenologically viable model given by Eq. (22). In order to proceed we will assume that the distribution of values of W_0 is uniform over the landscape and dense enough in our region of interest and that there are many minima nearby in field space to our inflationary inflection point.

We numerically evolve a large number of inflationary trajectories assuming a random initial condition for the fields near the inflection point in a potential generated by choosing a random value for W_0 within the tiny area compatible with more than 60 e-folds.

We plot in Fig. (7) the amplitude of scalar perturbations found 60 e-folds before the end

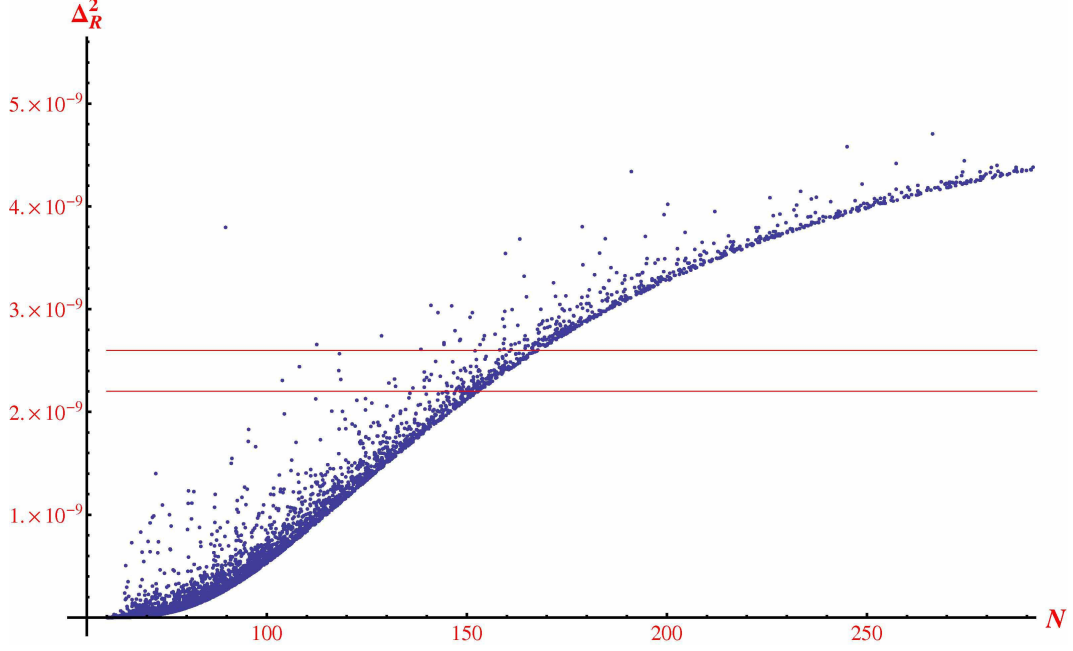


FIG. 7: Values for the amplitude of perturbations 60 e-folds before the end of inflation as a function of the total number of e-folds for the simulated trajectories in our mini-landscape of accidental inflation.

of inflation, on a run of 6000 different realizations together with the narrow band of the 2σ deviation from the observed value [1]. We see that this imposes a pretty strong constraint on the possible trajectories and allows us to discard many of them. We then proceed to calculate the spectral index predicted in this case and we show our results in Fig. (8) as well as the 2σ experimental band observed by WMAP.

We see on these two figures what seems to be a strong dependence of the observables on the number of e-folds together with some scattered points around it. This is again a manifestation of the fact that the most important effect that one introduces by changing the W_0 is to modify the slope of the near-inflection point. Most of the trajectories for each individual potential are close to the attractor solutions given by the single field slow roll conditions. Assuming these two effects one can account for the general dependence of these observables with the number of e-folds. We show how this occurs in the Appendix for a single field toy model.

Finally, the distribution on the number of e-folds in this case is again well described by a $1/N^3$ dependence reinforcing the idea that we can think of this landscape as being dominated

by a flat scanning of the first derivative of the inflection point inflationary potential.

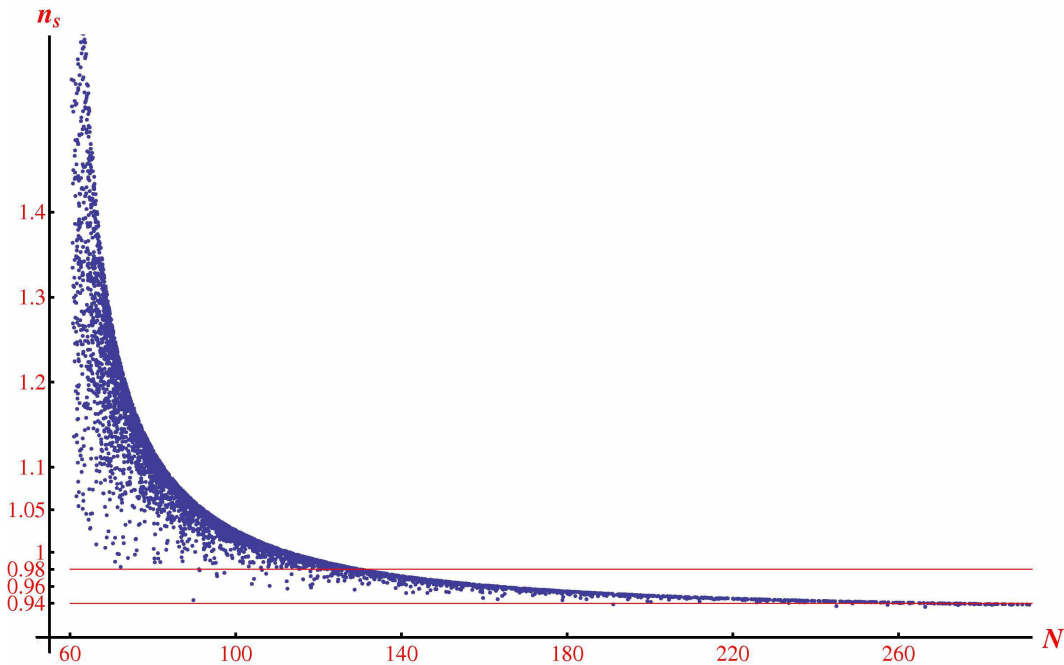


FIG. 8: Values of the spectral index (n_s) versus the number of e-folds for our mini-landscape.

We conclude that only 4% of all our trajectories are compatible with the current observational constraints. The main reason for this is that most of the trajectories have a small number of e-folds and a blue spectrum, as one would expect for an inflection point [27–29]. The results for those viable cases are highly peaked around the attractor solution with $N_{efolds} = 160$, $n_s = 0.96$ and $\Delta_{\mathcal{R}} = 2.5 \times 10^{-9}$, but there are a very small number of trajectories that correspond to the edges of the basin of attraction of this solution. An example of this would be a trajectory that started far away from the inflection point and reached the 60 e-folds before the end of inflation mark at the end of the slow roll region having undergone a small number of e-folds. These are interesting solutions where one may be able to observe some curvature. On the other hand, they are highly subdominant.

One could of course imagine a curvaton type scenario where the cosmological perturbations are generated by a different field not related to the inflaton. This is certainly a possibility that one could study in a string theory setup, see for example [44]. Following these ideas one decouples the distribution of the number of e-folds from the other observables related to the perturbations which can have an important effect on the overall predictions on the observable parameters in this landscape.

It is also important to emphasize that these results are obtained assuming the same values of the parameters A, B, a and b . In practice, this means that we are exploring a particular sub-sector of the landscape with a fixed hidden field theory. One can imagine that these parameters could also be scanned over in different sectors of the landscape. Changing the scale of, for example, A and B , would directly affect the scale of inflation so in principle one can rescale the amplitude of the perturbations to include some trajectories and not others.

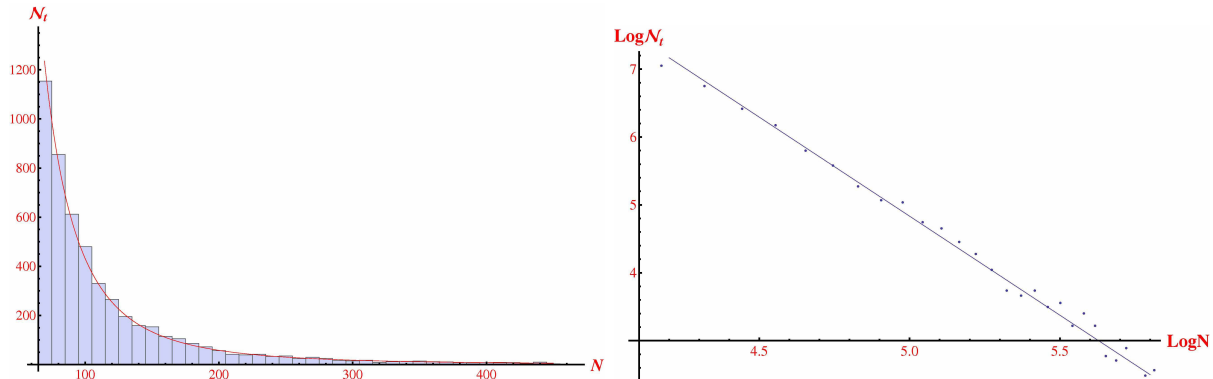


FIG. 9: Distribution of the number of e-folds in our simulated landscape. We show on the right hand figure the best fit of the data to a curve of the form $P(N) \sim N^\alpha$ with $\alpha = 2.92 \pm 0.06$.

VI. CONCLUSIONS

We focused in this paper on a particular model of inflation named Accidental Inflation [22] where the potential has to be finely tuned in order to give a substantial number of e-folds. We showed that this apparent fine tuning can be generically obtained by scanning the form of the potentials found in a very modestly small sector of the landscape generated by a family of six fluxes. Furthermore, the existence of a landscape in this model provides us with a theory of initial conditions for the inflationary period. Changing the fluxes from the cosmologically interesting one (the one that we have recently followed in our past history) one sees that the potential develops a local minimum nearby in field space. This is also a generic situation and we can show that there are many other vacua of this kind nearby. This suggests the scenario where the universe evolved from one of these vacua by tunneling out of it by a flux-changing instanton that triggers the transition to the daughter vacuum. This process gives us a natural way to select good initial conditions for our subsequent evolution

avoiding overshooting problems that normally occur in these type of models.

We have made some approximations in this paper and it would be interesting to investigate the extent of their validity. In particular, one can improve the calculation presented here by incorporating the actual Kahler moduli fields for the $P^4_{[1,1,1,6,9]}$ manifold. This would also allow us to explore other kind of models like the Large Volume Scenario [45] and possibly find minima of the complete supergravity potential and not rely on the KKLT type of constructions. Another interesting point would be to incorporate the dependence of the parameters A, B, \dots on the complex structure moduli. This is important since it would likely affect the conclusions about the distributions of the scale on inflation in a particular model.

We have also neglected other possible corrections to the potential that could be important for inflation. One can try to repeat the calculation that we have performed here taking into account those other terms estimated in [46].

Finally, one would like to extend this kind of arguments to other models of the string cosmology in order to be able to draw more generic conclusions that in combination with a measure would lead to a prediction of the inflationary observables in the string landscape.

VII. ACKNOWLEDGEMENTS

It is a pleasure to especially thank Per Berglund for his help with several important technical points. We would also like to thank Ken Olum, Delia Schwartz-Perlov, Mike Salem, Alex Vilenkin and Ben Shlaer for many useful discussions. J.J.B.-P. is supported in part by the National Science Foundation under the grant, PHY-0969910. M. G-R is supported by the Ramón y Cajal programme through the grant RYC-2009-04433. K.M. was supported in part by the John F. Burlingame Graduate Fellowship at Tufts.

Appendix A: Equations of motion for $N = 1$ supergravity

Starting with the $\mathcal{N} = 1$ supergravity action

$$S = - \int d^4x \sqrt{-g} \left[\frac{1}{2} R + K_{I\bar{J}} \partial_\mu \Phi^I \partial^\mu \Phi^{\bar{J}} + V(\Phi^M, \Phi^{\bar{M}}) \right] \quad (\text{A1})$$

and assuming that the universe is described by a FRW ansatz given by

$$ds^2 = -dt^2 + a(t)^2 d\Omega_k^2, \quad (\text{A2})$$

one can obtain the equations of motion for the moduli fields and the metric

$$\ddot{\phi}^i + 3 \left(\frac{\dot{a}}{a} \right) \dot{\phi}^i + \Gamma_{jk}^i \dot{\phi}^j \dot{\phi}^k + \mathbf{G}^{ij} \frac{\partial V}{\partial \phi^j} = 0 , \quad (\text{A3})$$

$$\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{1}{3} \left(\frac{1}{2} \mathbf{G}^{ij} \dot{\phi}^i \dot{\phi}^j + V \right) . \quad (\text{A4})$$

Note that $k = 0, \pm 1$ parametrizes the spatial curvature of the $3d$ part of the manifold, Γ_{jk}^i are the Christoffel symbols for the \mathbf{G}^{ij} metric in field space and ϕ^i denote the real components of the chiral fields such that

$$K_{I\bar{J}} \partial_\mu \Phi^I \partial^\mu \Phi^{\bar{J}} = \frac{1}{2} \mathbf{G}^{ij} \partial \phi^i \partial \phi^j . \quad (\text{A5})$$

Taking a single complex scalar field $\Phi = X + iY$ we arrive at the system of equations of the form,

$$\begin{aligned} \ddot{X} &= -3\dot{X} \frac{\dot{a}}{a} + \frac{\dot{X}^2 - \dot{Y}^2}{X} - \frac{2X^2 V_X}{3} , \\ \ddot{Y} &= -3\dot{Y} \frac{\dot{a}}{a} + \frac{2\dot{X}\dot{Y}}{X} - \frac{2X^2 V_Y}{3} , \\ \left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} &= \frac{\dot{X}^2 + \dot{Y}^2}{4X^2} + \frac{V}{3} . \end{aligned} \quad (\text{A6})$$

Once we have obtained the field trajectories we can calculate the slow roll parameters at any point using the general expressions for a 2 dimensional potential,

$$\epsilon = \frac{1}{2} \left(\frac{\mathbf{G}^{ij} \partial_i V \partial_j V}{V^2} \right) , \quad (\text{A7})$$

while η is defined as the most-negative eigenvalue of the matrix:

$$N_j^i = \frac{\mathbf{G}^{ik} (\partial_k \partial_j V - \Gamma_{kj}^l \partial_l V)}{V} . \quad (\text{A8})$$

1. Initial conditions for bubble universes.

As we discussed in the main part of the text, we are interested in studying the evolution of the fields in the interior of a bubble universe that forms as a consequence of a tunneling process. In order to do this, it is important to realize that the geometry of the bubble

interior is actually described by an infinite open universe [47]. On the other hand, the Big Bang of this open universe (the lightcone surface emanating from the nucleation center) is perfectly smooth and could be thought of as a piece of a Milne universe¹⁰. These constraints dictate that the initial conditions for this type of geometry should be,

$$a(t) = t + \mathcal{O}(t^3) \quad (\text{A9})$$

and

$$\begin{aligned} X(t) &= X_0^i + \mathcal{O}(t^2) , \\ Y(t) &= Y_0^i + \mathcal{O}(t^2) , \end{aligned} \quad (\text{A10})$$

where X_0^i and Y_0^i are the “exit point” in field space for the instanton that mediates between the parent and daughter vacua. Taking these initial conditions and the equations of motion (A6) with $k = -1$, one can obtain the subsequent evolution for the fields in the daughter bubble universe.

Appendix B: Analytical Estimates of Probability distributions.

We argued in the main part of the text that due to the nature of the attractor solution the results of our simulated landscape are quite insensitive to the initial conditions for the fields. This allows the possibility of understanding the results in terms of a much simpler single field inflation toy model¹¹.

We start by assuming that all the realizations of our landscape can locally be written around the inflection point as an expansion of the form

$$V \approx V_0 (1 - \lambda_1 \phi - \lambda_3 \phi^3) , \quad (\text{B1})$$

where ϕ denotes the canonically normalized field and the parameters of this expansion will be varying over our landscape. We can now calculate all the observables in terms of these parameters in a standard way. From the potential we get the slow roll inflation parameters,

$$\epsilon = \frac{1}{2} \left(\frac{V'}{V} \right)^2 \approx \frac{1}{2} (\lambda_1 + 3\lambda_3 \phi^2)^2 \quad (\text{B2})$$

¹⁰ Of course the whole history of the bubble interior is not describe by a Milne universe, only the early stages. Soon after the bubble formation the scale factor would evolve differently with time depending on the matter content inside of the bubble.

¹¹ In this Appendix we follow closely the discussion on [28] and [19].

and

$$\eta = \left(\frac{V''}{V} \right) \approx -6\lambda_3\phi \quad (\text{B3})$$

as well as the total number of efolds

$$N_{total}(\lambda_1, \lambda_3) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\epsilon}} d\phi = \frac{\pi}{\sqrt{3\lambda_1\lambda_3}} . \quad (\text{B4})$$

From this relation we get the expression for the η parameter at the CMB scale, assuming that the end of inflation is given by $\eta(\phi_{end}) = -1$,

$$\eta_{CMB}(\lambda_1, \lambda_3) = \frac{2\pi}{N_{total}} \left[\tan \left[\frac{\pi N_{CMB}}{N_{total}} - \arctan \left[\frac{N_{total}}{2\pi} \right] \right] \right] , \quad (\text{B5})$$

which is function of only the first and the third derivative of the potential. The spectral index can then be approximated by

$$n_s(\lambda_1, \lambda_3) = 1 + 2\eta_{CMB}(\lambda_1, \lambda_3) . \quad (\text{B6})$$

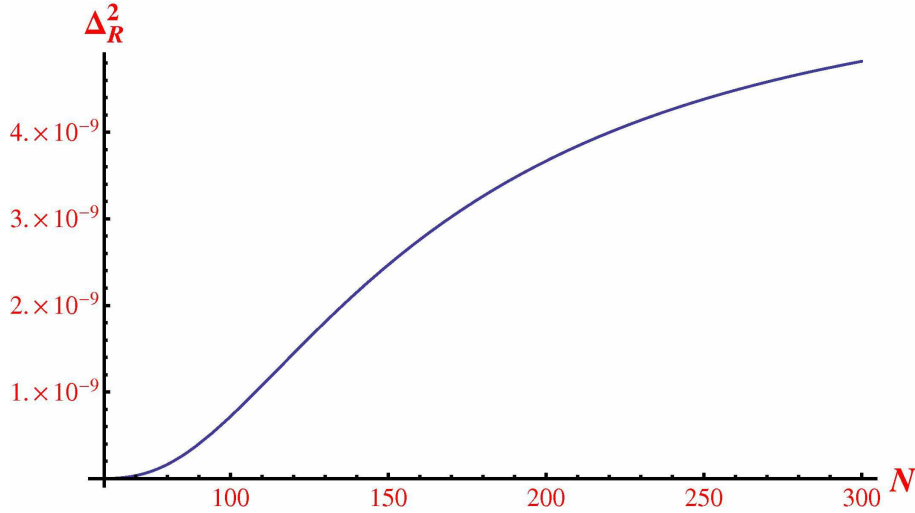


FIG. 10: Amplitude of perturbations versus the number of e-folds in a one dimensional landscape.

Using the relations in Eqs. (B2) and (B3) we can approximate the position in field space 60 e-folds before the end of inflation as well as ϵ by the expressions

$$\phi_{CMB} = -\frac{\eta_{CMB}}{6\lambda_3} , \quad (\text{B7})$$

$$\epsilon_{CMB} = \frac{1}{2} (\lambda_1 + 3\lambda_3\phi_{CMB}^2)^2 . \quad (\text{B8})$$

Using this information we can obtain the scalar power spectrum

$$\Delta_{\mathcal{R}}^2(V_0, \lambda_1, \lambda_3) = \frac{1}{24\pi^2} \frac{V}{\epsilon} \bigg|_{CMB} . \quad (\text{B9})$$

These equations will hold for any inflection point model so they will apply for each of our realizations. We can then use these relations to estimate the distribution of values over the landscape assuming that one varies W_0 in a uniform way over the area that leads to more than 60 e-folds.

Changing the superpotential induces small variations on the parameters of the inflection point potential which mostly do not change things significantly, except the variation of λ_1 . One can then model the real landscape by assuming that one only scans uniformly over this one parameter, λ_1 . Taking into account that $N_{total} \sim 1/\sqrt{\lambda_1}$ one arrives at a distribution on the number of e-folds of the form,

$$P(N) \sim \frac{1}{N^3} . \quad (\text{B10})$$

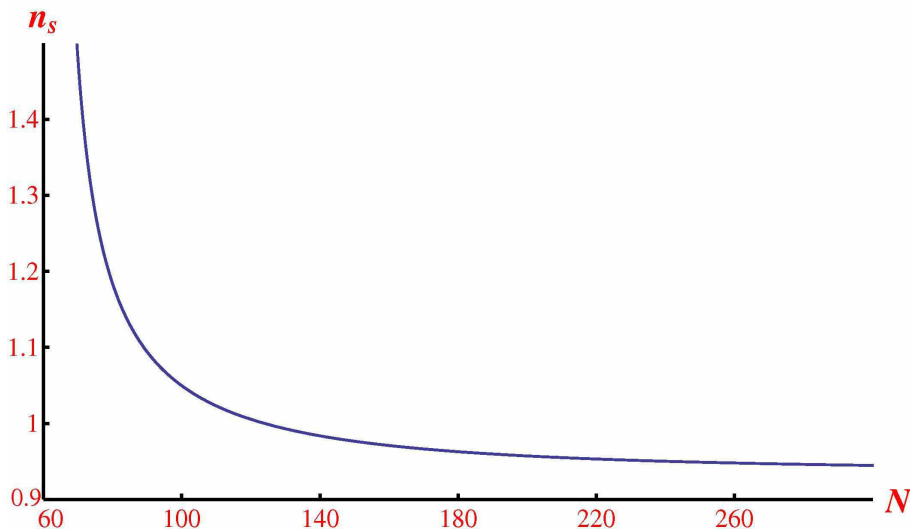


FIG. 11: n_s versus the number of e-folds, N , in a one dimensional landscape.

Assuming this dependence of the number of e-folds with the variable being scanned over the landscape, λ_1 , and using the relations found earlier in B9 and B6 one can find the the amplitude of perturbations and the n_s parameter as a function of the number of e-folds. We plot these functions in Figs. (10) and (11).

Comparing these figures to the ones we found in our random landscape we can infer that most of the simulated trajectories closely follow this analytic form. This is due to the existence of the attractor solution. There are however some special cases where the

trajectory never enters the attractor solution, but they are statistically not very significant.

-
- [1] E. Komatsu *et al.* [WMAP Collaboration], “Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation,” *Astrophys. J. Suppl.* **192**, 18 (2011).
 - [2] F. Quevedo, “Lectures on string/brane cosmology,” *Class. Quant. Grav.* **19** (2002) 5721;
A. Linde, “Inflation and string cosmology,” eConf **C040802**, L024 (2004) [*J. Phys. Conf. Ser.* **24**, 151 (2005 PTPSA,163,295-322.2006)];
J. M. Cline, “String cosmology,” arXiv:hep-th/0612129;
C. P. Burgess, “Lectures on Cosmic Inflation and its Potential Stringy Realizations,” *PoS P2GC*, 008 (2006) [*Class. Quant. Grav.* **24**, S795 (2007)];
R. Kallosh, “On Inflation in String Theory,” *Lect. Notes Phys.* **738**, 119 (2008);
L. McAllister and E. Silverstein, “String Cosmology: A Review,” *Gen. Rel. Grav.* **40** (2008) 565;
D. Baumann and L. McAllister, “Advances in Inflation in String Theory,” *Ann. Rev. Nucl. Part. Sci.* **59**, 67 (2009);
M. Cicoli and F. Quevedo, “String moduli inflation: An overview,” *Class. Quant. Grav.* **28**, 204001 (2011);
C. P. Burgess and L. McAllister, “Challenges for String Cosmology,” *Class. Quant. Grav.* **28**, 204002 (2011).
 - [3] P. G. O. Freund and M. A. Rubin, “Dynamics of Dimensional Reduction,” *Phys. Lett. B* **97**, 233 (1980).
 - [4] M. R. Douglas and S. Kachru, “Flux compactification,” *Rev. Mod. Phys.* **79**, 733 (2007).
 - [5] S. B. Giddings, S. Kachru and J. Polchinski, “Hierarchies from fluxes in string compactifications,” *Phys. Rev. D* **66**, 106006 (2002).
 - [6] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, “De Sitter vacua in string theory,” *Phys. Rev. D* **68**, 046005 (2003).
 - [7] R. Bousso and J. Polchinski, “Quantization of four form fluxes and dynamical neutralization of the cosmological constant,” *JHEP* **0006**, 006 (2000).
 - [8] L. Susskind, “The anthropic landscape of string theory,” (2003), arXiv:hep-th/0302219.

- [9] D. Lust, “String Landscape and the Standard Model of Particle Physics,” arXiv:0707.2305 [hep-th].
- [10] J. J. Blanco-Pillado, D. Schwartz-Perlov and A. Vilenkin, “Transdimensional Tunneling in the Multiverse,” JCAP **1005**, 005 (2010).
- [11] G. R. Dvali and S. H. H. Tye, “Brane inflation,” Phys. Lett. B **450**, 72 (1999);
 G. R. Dvali, Q. Shafi and S. Solganik, “D-brane inflation,” hep-th/0105203.
 S. Kachru, R. Kallosh, A. Linde, J. M. Maldacena, L. P. McAllister and S. P. Trivedi, “Towards inflation in string theory,” JCAP **0310**, 013 (2003);
 C. P. Burgess, J. M. Cline, H. Stoica and F. Quevedo, “Inflation in realistic D-brane models,” JHEP **0409**, 033 (2004);
 E. Silverstein and D. Tong, “Scalar speed limits and cosmology: Acceleration from Dcceleration,” Phys. Rev. D **70**, 103505 (2004);
 M. Alishahiha, E. Silverstein and D. Tong, “DBI in the sky,” Phys. Rev. D **70**, 123505 (2004);
 X. Chen, “Inflation from warped space,” JHEP **0508**, 045 (2005);
 D. Baumann, A. Dymarsky, I. R. Klebanov, L. McAllister and P. J. Steinhardt, “A Delicate Universe,” Phys. Rev. Lett. **99** (2007) 141601;
 S. Panda, M. Sami and S. Tsujikawa, “Prospects of inflation in delicate D-brane cosmology,” Phys. Rev. D **76**, 103512 (2007);
 D. A. Easson, R. Gregory, D. F. Mota, G. Tasinato and I. Zavala, “Spinflation,” JCAP **0802**, 010 (2008);
 D. Baumann, A. Dymarsky, S. Kachru, I. R. Klebanov and L. McAllister, “Holographic Systematics of D-brane Inflation,” JHEP **0903**, 093 (2009).
- [12] J.J. Blanco-Pillado, C.P. Burgess, J.M. Cline, C. Escoda, M. Gomez-Reino, R. Kallosh, A. Linde, F. Quevedo, “Racetrack inflation,” JHEP **0411** (2004) 063;
 J.J. Blanco-Pillado, C.P. Burgess, J.M. Cline, C. Escoda, M. Gomez-Reino, R. Kallosh, A. Linde, F. Quevedo, “Inflating in a better racetrack,” JHEP **0609** (2006) 002;
 R. Holman and J. A. Hutasoit, “Systematics of moduli stabilization, inflationary dynamics and power spectrum,” JHEP **0608** (2006) 053;
 J. P. Conlon and F. Quevedo, “Kaehler moduli inflation,” JHEP **0601** (2006) 146.
 J. R. Bond, L. Kofman, S. Prokushkin and P. M. Vaudrevange, “Roulette inflation with Kaehler moduli and their axions,” Phys. Rev. D **75** (2007) 123511.

- Z. Lalak, G. G. Ross and S. Sarkar, “Racetrack inflation and assisted moduli stabilisation,” Nucl. Phys. B **766** (2007) 1;
- B. de Carlos, J. A. Casas, A. Guarino, J. M. Moreno and O. Seto, “Inflation in uplifted supergravities,” JCAP **0705** (2007) 002;
- T. W. Grimm, “Axion Inflation in Type II String Theory,” Phys. Rev. D **77**, 126007 (2008);
- S. Dimopoulos, S. Kachru, J. McGreevy and J. G. Wacker, “N-flation,” JCAP **0808**, 003 (2008);
- R. Kallosh, N. Sivanandam and M. Soroush, “Axion Inflation and Gravity Waves in String Theory,” Phys. Rev. D **77**, 043501 (2008);
- V. Balasubramanian, P. Berglund, R. Jimenez, J. Simon and L. Verde, “Topology from Cosmology,” JHEP **0806**, 025 (2008);
- H. X. Yang and H. L. Ma, “Two-field Kahler moduli inflation on large volume moduli stabilization,” JCAP **0808**, 024 (2008);
- A. Misra and P. Shukla, “Large Volume Axionic Swiss-Cheese Inflation,” Nucl. Phys. B **800** (2008) 384;
- M. Cicoli, C. P. Burgess and F. Quevedo, “Fibre Inflation: Observable Gravity Waves from IIB String Compactifications,” JCAP **0903** (2009) 013;
- E. Silverstein and A. Westphal, “Monodromy in the CMB: Gravity Waves and String Inflation,” Phys. Rev. D **78**, 106003 (2008)
- J. J. Blanco-Pillado, D. Buck, E. J. Copeland, M. Gomez-Reino and N. J. Nunes, “Kahler Moduli Inflation Revisited,” JHEP **1001**, 081 (2010);
- M. Cicoli, F. G. Pedro and G. Tasinato, “Poly-instanton Inflation,” JCAP **1112**, 022 (2011).
- [13] M. Bucher, A. S. Goldhaber and N. Turok, “An open universe from inflation,” Phys. Rev. D **52**, 3314 (1995);
- M. Sasaki, T. Tanaka and K. Yamamoto, “Euclidean vacuum mode functions for a scalar field on open de Sitter space,” Phys. Rev. D **51**, 2979 (1995);
- K. Yamamoto, M. Sasaki and T. Tanaka, “Large angle CMB anisotropy in an open universe in the one bubble inflationary scenario,” Astrophys. J. **455**, 412 (1995);
- M. Bucher and N. Turok, “Open inflation with arbitrary false vacuum mass,” Phys. Rev. D **52**, 5538 (1995);
- A. D. Linde, “Inflation with variable Omega,” Phys. Lett. B **351**, 99 (1995);

- J. Garriga, X. Montes, M. Sasaki and T. Tanaka, “Canonical quantization of cosmological perturbations in the one bubble open universe,” Nucl. Phys. B **513**, 343 (1998)
- J. Garriga, X. Montes, M. Sasaki and T. Tanaka, “Spectrum of cosmological perturbations in the one bubble open universe,” Nucl. Phys. B **551**, 317 (1999);
- A. D. Linde, M. Sasaki and T. Tanaka, “CMB in open inflation,” Phys. Rev. D **59**, 123522 (1999);
- J. Garcia-Bellido, J. Garriga and X. Montes, “Microwave background anisotropies in quasiopen inflation,” Phys. Rev. D **60**, 083501 (1999).
- [14] B. Freivogel, M. Kleban, M. Rodriguez Martinez and L. Susskind, “Observational consequences of a landscape,” JHEP **0603**, 039 (2006).
- [15] D. Yamauchi, A. Linde, A. Naruko, M. Sasaki and T. Tanaka, “Open inflation in the landscape,” Phys. Rev. D **84**, 043513 (2011).
- [16] A. Vilenkin, “A Measure of the multiverse,” J. Phys. A **40**, 6777 (2007);
- A. H. Guth, “Eternal inflation and its implications,” J. Phys. A **40**, 6811 (2007);
- B. Freivogel, “Making predictions in the multiverse,” Class. Quant. Grav. **28**, 204007 (2011);
- M. P. Salem, “Bubble collisions and measures of the multiverse,” JCAP **1201**, 021 (2012).
- [17] A. H. Guth and Y. Nomura, “What can the observation of nonzero curvature tell us?,” arXiv:1203.6876 [hep-th].
- [18] M. Tegmark, “What does inflation really predict?,” JCAP **0504**, 001 (2005);
- A. Aazami and R. Easther, “Cosmology from random multifield potentials,” JCAP **0603**, 013 (2006);
- R. Easther and L. McAllister, “Random matrices and the spectrum of N-flation,” JCAP **0605**, 018 (2006);
- S. -H. H. Tye, J. Xu and Y. Zhang, “Multi-field Inflation with a Random Potential,” JCAP **0904**, 018 (2009);
- D. Battefeld and T. Battefeld, “Multi-Field Inflation on the Landscape,” JCAP **0903**, 027 (2009);
- J. Frazer and A. R. Liddle, “Exploring a string-like landscape,” JCAP **1102**, 026 (2011);
- D. Battefeld, T. Battefeld and S. Schulz, “On the Unlikeliness of Multi-Field Inflation: Bounded Random Potentials and our Vacuum,” arXiv:1203.3941 [hep-th].
- [19] N. Agarwal, R. Bean, L. McAllister and G. Xu, “Universality in D-brane Inflation,” JCAP

- 1109**, 002 (2011).
- [20] M. Dias, J. Frazer and A. R. Liddle, “Multifield consequences for D-brane inflation,” arXiv:1203.3792 [astro-ph.CO].
 - [21] J. M. Cline and H. Stoica, “Multibrane inflation and dynamical flattening of the inflaton potential,” Phys. Rev. D **72**, 126004 (2005); J. M. Cline, L. Hoi and B. Underwood, “Dynamical Fine Tuning in Brane Inflation,” JHEP **0906**, 078 (2009); L. Hoi and J. M. Cline, “How Delicate is Brane-Antibrane Inflation?,” Phys. Rev. D **79**, 083537 (2009).
 - [22] A. Linde and A. Westphal, “Accidental Inflation in String Theory,” JCAP **0803** (2008) 005.
 - [23] P. Candelas and X. de la Ossa, “Moduli Space Of Calabi-yau Manifolds,” Nucl. Phys. B **355**, 455 (1991).
 - [24] S. Gukov, C. Vafa and E. Witten, “CFT’s from Calabi-Yau four folds,” Nucl. Phys. B **584**, 69 (2000) [Erratum-ibid. B **608**, 477 (2001)].
 - [25] R. Kallosh and A. D. Linde, “Landscape, the scale of SUSY breaking, and inflation,” JHEP **0412**, 004 (2004). J. J. Blanco-Pillado, R. Kallosh and A. D. Linde, “Supersymmetry and stability of flux vacua,” JHEP **0605**, 053 (2006).
 - [26] R. Brustein and P. J. Steinhardt, “Challenges for superstring cosmology,” Phys. Lett. B **302**, 196 (1993).
 - [27] R. Allahverdi, K. Enqvist, J. Garcia-Bellido and A. Mazumdar, “Gauge invariant MSSM inflaton,” Phys. Rev. Lett. **97**, 191304 (2006);
R. Allahverdi, K. Enqvist, J. Garcia-Bellido, A. Jokinen and A. Mazumdar, “MSSM flat direction inflation: Slow roll, stability, fine tuning and reheating,” JCAP **0706**, 019 (2007);
N. Itzhaki and E. D. Kovetz, “Inflection Point Inflation and Time Dependent Potentials in String Theory,” JHEP **0710**, 054 (2007);
R. Allahverdi, B. Dutta and A. Mazumdar, “Attraction towards an inflection point inflation,” Phys. Rev. D **78**, 063507 (2008);
M. Spalinski, “Initial conditions for small field inflation,” Phys. Rev. D **80**, 063529 (2009);
K. Enqvist, A. Mazumdar and P. Stephens, “Inflection point inflation within supersymmetry,” JCAP **1006**, 020 (2010);
S. Hotchkiss, A. Mazumdar and S. Nadathur, “Inflection point inflation: WMAP constraints and a solution to the fine-tuning problem,” JCAP **1106**, 002 (2011).
 - [28] D. Baumann, A. Dymarsky, I. R. Klebanov and L. McAllister, “Towards an Explicit Model

- of D-brane Inflation,” JCAP **0801** (2008) 024.
- [29] J. P. Conlon, R. Kallosh, A. D. Linde and F. Quevedo, “Volume Modulus Inflation and the Gravitino Mass Problem,” JCAP **0809**, 011 (2008);
M. Badziak and M. Olechowski, “Volume modulus inflation and a low scale of SUSY breaking,” JCAP **0807**, 021 (2008);
M. Badziak and M. Olechowski, “Volume modulus inflection point inflation and the gravitino mass problem,” JCAP **0902**, 010 (2009).
- [30] P. Candelas, A. Font, S. H. Katz and D. R. Morrison, “Mirror symmetry for two parameter models. 2.,” Nucl. Phys. B **429**, 626 (1994)
- [31] F. Denef, M. R. Douglas and B. Florea, “Building a better racetrack,” JHEP **0406**, 034 (2004).
- [32] G. Curio and V. Spillner, “On the modified KKLT procedure: A Case study for the P(11169) [18] model,” Int. J. Mod. Phys. A **22**, 3463 (2007).
- [33] A. Giryavets, S. Kachru and P. K. Tripathy, “On the taxonomy of flux vacua,” JHEP **0408**, 002 (2004);
A. Giryavets, S. Kachru, P. K. Tripathy and S. P. Trivedi, “Flux compactifications on Calabi-Yau threefolds,” JHEP **0404**, 003 (2004);
J. P. Conlon and F. Quevedo, “On the explicit construction and statistics of Calabi-Yau flux vacua,” JHEP **0410**, 039 (2004)
A. Giryavets, “New attractors and area codes,” JHEP **0603**, 020 (2006).
- [34] M. R. Douglas, “Statistical analysis of the supersymmetry breaking scale,” hep-th/0405279.
- [35] S. Kachru, J. Pearson and H. L. Verlinde, “Brane / flux annihilation and the string dual of a nonsupersymmetric field theory,” JHEP **0206**, 021 (2002).
- [36] A. R. Frey, M. Lippert and B. Williams, “The Fall of stringy de Sitter,” Phys. Rev. D **68**, 046008 (2003).
- [37] J. J. Blanco-Pillado, D. Schwartz-Perlov and A. Vilenkin, “Quantum Tunneling in Flux Compactifications,” JCAP **0912**, 006 (2009).
- [38] M. C. Johnson and M. Larfors, “Field dynamics and tunneling in a flux landscape,” Phys. Rev. D **78**, 083534 (2008);
P. Ahlqvist, B. R. Greene, D. Kagan, E. A. Lim, S. Sarangi and I-S. Yang, “Conifolds and Tunneling in the String Landscape,” JHEP **1103**, 119 (2011).

- [39] A. R. Brown and A. Dahlen, “Giant Leaps and Minimal Branes in Multi-Dimensional Flux Landscapes,” *Phys. Rev. D* **84**, 023513 (2011).
- [40] M. Kleban, K. Krishnaiyengar and M. Porrati, “Flux Discharge Cascades in Various Dimensions,” *JHEP* **1111**, 096 (2011).
- [41] K. Dutta, P. M. Vaudrevange and A. Westphal, “The Overshoot Problem in Inflation after Tunneling,” *JCAP* **1201**, 026 (2012).
- [42] P. M. Vaudrevange and A. Westphal, “A Toy Model For Single Field Open Inflation,” *arXiv:1205.1663* [hep-th].
- [43] A. De Simone and M. P. Salem, “The distribution of Ω_k from the scale-factor cutoff measure,” *Phys. Rev. D* **81**, 083527 (2010).
- [44] C. P. Burgess, M. Cicoli, M. Gomez-Reino, F. Quevedo, G. Tasinato and I. Zavala, “Non-standard primordial fluctuations and nongaussianity in string inflation,” *JHEP* **1008** (2010) 045;
M. Cicoli, G. Tasinato, I. Zavala, C. P. Burgess and F. Quevedo, “Modulated Reheating and Large Non-Gaussianity in String Cosmology,” *JCAP* **1205**, 039 (2012).
- [45] V. Balasubramanian, P. Berglund, J. P. Conlon and F. Quevedo, “Systematics of moduli stabilisation in Calabi-Yau flux compactifications,” *JHEP* **0503**, 007 (2005).
- [46] J. P. Conlon, F. Quevedo and K. Suruliz, “Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking,” *JHEP* **0508**, 007 (2005);
S. S. AbdusSalam, J. P. Conlon, F. Quevedo and K. Suruliz, “Scanning the Landscape of Flux Compactifications: Vacuum Structure and Soft Supersymmetry Breaking,” *JHEP* **0712**, 036 (2007);
M. Cicoli, J. P. Conlon and F. Quevedo, “Systematics of String Loop Corrections in Type IIB Calabi-Yau Flux Compactifications,” *JHEP* **0801**, 052 (2008).
- [47] S. R. Coleman and F. De Luccia, “Gravitational Effects on and of Vacuum Decay,” *Phys. Rev. D* **21**, 3305 (1980).